Traditional and IS-enabled Customer Acquisition for an Internet Retailer: Why New Buyer Acquisition Varies over Geographies and by Method

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Abstract

We provide new insights into the efficacy of offline word-of-mouth (WOM) and magazine advertising (traditional acquisition methods) compared to online WOM and online search (IS-enabled acquisition methods). We study how three key factors—inter-customer proximity, price benefits, and convenience benefits—affect the performance of each acquisition method. We estimate a multivariate NBD model over new buyer counts at all US residential zip codes. All else equal, favorable local conditions for the three factors benefit WOM acquisitions most. WOM acquisitions rely on customer-to-customer communication and respond to factors that facilitate interaction. Second, traditional offline WOM is more effective than IS-enabled online WOM; since offline WOM recipients are co-located with senders they share the same benefits of shopping online. Third, traditional acquisition modes still matter for online retailers and are complementary—WOM acquisitions dominate in high-performing spatially-clustered markets and magazine advertising acquisitions dominate in low-performing spatially-dispersed markets. IS-enabled methods provide a relatively constant percentage contribution to customer acquisition. Lastly, we provide some evidence that using the model-based predictions to target specific markets leads to improvements in actual click-to-order rates, up to a factor of two.

Keywords: *Count Model*; *Internet Retailing*; *Search*; *Spatial Analysis*; *Word-of-Mouth*

Traditional retailers with physical stores have relatively small trading areas (Fotheringham 1988; Huff 1964), however customer acquisition at these firms is quite straightforward as their customer acquisition efforts are bounded by geography. Internet retailers, on the other hand, have ample markets to serve (Bell and Song 2007), yet it is not clear where they should focus their acquisition efforts. Thus, the two institutional setups—traditional and online retailing—generate different cost-benefit trade-offs for retailers. Consumers also face contrasting cost-benefit trade-offs. Consumers can easily discover and visit local offline stores, but travel costs preclude inspection of geographically distant offline alternatives. Conversely, Internet retail alternatives for many consumer products are plentiful, yet consumers may not know how to initially “find” the store that best suits their needs. These trade-offs underlie the conceptual basis for our research on customer acquisition for an Internet retailer.

Recent studies (e.g., Anderson at al. 2009; Forman, Ghose, and Goldfarb 2009) show that the value to consumers of shopping online varies substantially across local markets as a function of the relative price and convenience of local offline options. In addition to these critical factors, proximity among target customers—which facilitates social influence—also plays a key role (Choi, Hui, and Bell 2010). We build on these general findings by directly addressing the unexplored issue of how customers discover online retailers to begin with. We decompose total customer acquisitions into acquisitions from four different modes— offline word-of-mouth (WOM), online WOM, online search, and magazine advertising—and compare and contrast the relative demand effects over space. In particular, we show how and why the efficacy of each discovery process varies over space, and therefore why understanding this disaggregate demand process is central to the Internet retailer’s customer acquisition efforts. Two contrasts are relevant to researchers in marketing and IS (see Figure 1). Specifically, the comparison between traditional methods (offline WOM and advertising) and new IS-enabled methods (online WOM and online search) and between customer-generated acquisitions, i.e., those from WOM sources, and those elicited by the firm though online keyword search and magazine advertising.

[Insert Figure 1 about here]

We formulate a multivariate NBD model to analyze these four acquisition processes and calibrate it on data from a leading Internet retailer, Childcorp.com.[1] The acquisition rate for each process depends on locally-defined variables (discussed subsequently) that affect the utility of shopping online. To accurately capture the demand effects of the local variables, we control for the baselines acquisition rates through multivariate regional random effects (Gueorguieva 2001; Thum 1997) and also allow for specification error that could arise from "misclassification" of new customers. Evaluation of the likelihood over the multivariate random effects is computationally demanding, so we fit pair-wise bivariate models separately and compute the parameter estimates and their sampling variation for the full multivariate model (Fieuws and Verbeke 2006; Fieuws et al. 2006).

Our empirical findings provide new insights into the efficacy of traditional and IS-enabled customer acquisition methods. First, different acquisition methods respond differently to variation in the key variables of inter-customer proximity, price benefits, and convenience benefits. WOM—offline WOM in particular—shows the strongest response to these three factors. WOM is an interdependent process at the individual level, so it benefits disproportionately from factors that facilitate interaction among target customers. In an aggregated zip level demand model, this shows up in larger demand estimates and associated marginal effects (Becker and Murphy 2000; Glaeser, Sacerdote, and Scheinkman 1996). Our marginal effects imply that while

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[1] For reasons of confidentiality we refer to this leading Internet retailer by the *nom de plume*, "Childcorp.com". Acquisition information is collected during customer registration. We provide more details in the Data section.

WOM customers account for about one third of the total number of customers in “average” zip codes, they account for one half of the “lift” when conditions are improved. Second, traditional offline WOM appears to be more effective than IS-enabled online WOM. Offline WOM recipients are co-located with senders so they share the same price and convenience benefits of shopping online, whereas online recipients who are spatially separated from senders may not. (WOM effectiveness in new buyer acquisition is enhanced by spatial co-location of senders and receivers.) Third, acquisition modes contribute differently to the overall customer base. Offline WOM is very effective in high-performing markets and generates new buyers that are spatially clustered. Conversely, online search and magazine advertising increase in relative effectiveness as the “average yield” of zip codes decline. Magazine advertising is key in reaching more isolated geographic markets, that individually have few model-based expected buyers, but collectively many buyers (see Figure 5 and later discussion).

We conclude that traditional marketing efforts are still important to firms in the new economy and provide some evidence that geo-targeting will be vital to the success of Internet retailers, especially those with limited resources. We use model-based predictions to identify zip codes that should have higher propensities to generate new buyers and using city level data from a separate source (Coremetrics.com), we find that these zip codes have very good actual click to conversion rates.

The paper is organized as follows. The next section summarizes relevant findings from the literature. The following two sections describe the data and the empirical model. We then report the empirical findings. The paper concludes with a summary of the key findings and implications for managers and for future research.

BACKGROUND LITERATURE

Prior studies highlight trade-offs that lead customers to shop online instead of offline. While the outcome of these trade-offs is critical to an Internet retailer, i.e., the customer either shops online or not, equally important is an understanding of the customer's "path" to shopping online. Thus, our objective is to tease out across-location differences in how customers arrive at an Internet retailer through qualitatively distinct acquisition paths. Next, we briefly review key drivers for shopping online, followed by a summary of insights on the multiple ways firms acquire customers, and the corresponding implications for an Internet retailer.

Key Drivers for Shopping Online

Consumer benefits from using online retailers include lower prices (Anderson et al. 2009; Goolsbee 2000) and greater convenience (Brynjolfsson and Smith 2000; Forman, Ghose, and Goldfarb 2009; Keeney 1999). The difference between online sales tax rates (typically zero) and offline sales tax rates can create strong price-based incentives for shopping online.[2] As evidence, Goolsbee (2000) finds that if Internet retail transactions were taxed at average offline rates (about 8%), online demand would decline by 24%. Similarly, Anderson et al. (2009) show that when an Internet retailer opens physical stores and collects sales tax in locations where it previously did not, Internet sales suffer.

The importance of convenience is highlighted in a *Wall Street Journal* study (October 4, 2007) which evaluated several competing online sites, to which the firm supplying our data was

[2] Since Internet retailers collect no sales tax in locations where they have no retail presence, most locations enjoy tax-free shopping from online retailers. Sensitivity of online sales to local offline sales tax rates could mirror cross-border shopping observed for traditional stores; consumers might arbitrage across the "border" between the online stores and the local offline stores.

similar: “Getting an online discount doesn’t matter much if you have to pinch-hit with pricier (products) from the grocery store while you wait for your order to arrive.” For an Internet retailer, convenience works in two directions—through the shipping time proximity to customers (time distance) and through the physical travel distance between offline retail competitors and customers (travel distance). Prior research (Brynjolfsson and Smith 2000) finds that some customers pay a premium for faster shipping (reduced time distance). Similarly, reduced travel distance to offline alternatives makes the online retailer *less* attractive to customers (Forman, Ghose, and Goldfarb 2009).

Decisions to shop online are influenced by price and convenience factors; however, social influence from existing customers might also facilitate them. Since social influence can be hard to measure directly, prior studies often use physical proximity as a proxy for the propensity of customers to communicate with each other or observe each others’ behavior (e.g., Yang and Allenby 2003). Recent studies find that the trial behavior of Internet retail shoppers is driven by proximity to existing customers (Bell and Song 2007), and emulation among physically close customers is especially important in the early stages of online demand evolution (Choi, Hui, and Bell 2010). Our empirical model therefore follows the literature and includes variables that capture the price and convenience benefits of shopping online, as well as a measure of physical proximity among target customers.

Customer Acquisition Methods for Online Firms

Consumers find online retailers in four basic ways noted in the Introduction (see Figure 1). As shown in Figure 1, shoppers who respond to magazine advertising or use online search act relatively *independently* of other consumers. Those who are converted from magazine advertising are likely persuaded by the message content (e.g., Lodish et al. 1995) and those who discover online retailers through online search may be motivated by price and convenience benefits (e.g., Bakos 1997).

Alternatively, new buyers can interact socially with existing customers and find an online retailer through WOM. The ability of WOM to generate demand for local offline stores is well known. Katz and Lazarfeld (1955), in a classic study, find WOM is seven times as effective as magazine advertising and twice as effective as radio advertising. Recent work implies that the superiority of WOM holds for online retailers as well. Villanueva, Yoo, and Hanssens (2008) find that customers acquired via WOM add nearly twice as much value to long term equity as customers acquired by marketing efforts.[3]

Customers from WOM rely on the credible information from trusted sources, and thus these shoppers are operating somewhat *interdependently* (as shown in Figure 1). This interdependency among buyers creates a synergistic effect called a “social multiplier”. This has an important implication for empirical analysis. Factors generating positive social influence at the individual buyer level show larger aggregate demand coefficients for *interdependent* demand processes, than they do for *independent* demand processes (Becker and Murphy 2000; Glaeser, Sacerdote, and Scheinkman 1996). Our empirical model uses zip-level aggregate demand, so we expect demand coefficients on focal variables to be larger for WOM acquisitions than they are for acquisitions through online search and magazine advertising.

Our setting also permits a more nuanced examination of WOM acquisitions. While information technology helps customers connect and share information with geographically-dispersed and unacquainted others (Dellarocas 2003; Godes and Mayzlin 2009), the efficacy of

[3] Their definition of WOM is rather broad as it includes links from search engines and referrals from friends and colleagues. In this study, we distinguish both as online search and offline WOM, respectively. More details are given in the Data section.

acquisitions through offline and online WOM has not been examined. We compare the effects of local factors on "location-based referral" (offline WOM) and "IT-enabled referral" (online WOM). We expect that customers recruited through offline WOM, on average, experience greater benefits from shopping online than those acquired by online WOM do. When a customer tells a proximate neighbor about an online retailer, WOM is transmitted to a recipient who faces the *same* offline options as those faced by the sender, i.e., the recipient obtains approximately the *same relative benefit* from shopping online instead of offline. Conversely, online WOM may come from senders who are physically distant from recipients. Thus, we expect to see acquisitions through offline WOM respond more strongly to the key demand drivers of shopping online. Information on the physical locations of the information senders and information recipients could be helpful in exploring the effectiveness of online WOM through blogs and so forth (e.g., Dellarocas 2003; Dellarocas and Wood 2008).

DATA

Zip Level Cumulative Numbers of New Buyers at Childcorp.com

Childcorp.com is a *nom de plume* for a leading Internet retailer selling a large selection of name brand childrens products that are distributed nationally through various offline stores (all supermarkets, discount stores, and warehouse clubs). Childcorp.com product quality can be determined *ex ante*, i.e., the products possess few if any non-digital attributes (Lal and Sarvary 1999; Lynch and Ariely 2000), and prices are comparable to those at Wal-Mart. Shipping is free with orders over $49 (about 90% of all orders receive free shipping) and UPS ships from company warehouses located in both the eastern and western US. When individual shoppers register at Childcorp.com, they are asked: "How did you hear about our website?" Multiple responses are prevented through the use of a drop-down list and all the answers are classified into the four mutually exclusive and collectively exhaustive categories—offline WOM, online WOM, online search, and magazine advertising.[4]

Offline WOM includes personal referrals from friends, colleagues, or acquaintances, and accidental referrals from unacquainted people in local regions. *Online WOM* includes referrals through online message boards, blogs, and online communities. *Magazine advertising* includes ads in an affiliated magazine targeted at the customer group. *Online search* includes paid and organic keyword search from search engines, and connections from sponsored price comparison sites. Acquisitions through magazine advertising and online search come from buyer decisions that are largely free from direct influence by other customers. Conversely, WOM acquisitions result from interdependent buyer behavior. Interdependence, relative to independence, at the individual level leads to larger aggregate model demand coefficients (Becker and Murphy 2000; Glaeser, Sacerdote, and Scheinkman 1996).

For model estimation we obtained zip level counts of the number of new buyers acquired through each of the four processes, from the inception of the site in January 2005, through March 2008. Working at the zip level is not only practical in terms of data requirements (detailed individual level information is not collected and unavailable), but also managerially useful. Childcorp.com competitors are local offline retailers located in many zip codes throughout the

[4] About 31% of buyers did not answer the question. However we understand from management that the ordering behavior of this non-respondent group does not differ significantly from that of the respondent group; hence, we believe the data are relatively free of non-response bias. Moreover, we also have no reason to believe that individuals systematically distort their self-report and the model specification error at the zip level helps to account for both "imperfect memory" of consumers as well as the possibility that an individual was influenced in multiple ways (see the Empirical Model section for a detailed discussion).

United States; several other secondary data sources that inform the model specification are also collected at the zip code level.

Figure 1 shows the spatial variation in the number of new buyers acquired via the four acquisition processes and Table 1 presents the corresponding summary statistics. Together Figure 1 and Table 1 show the following. The number of new buyers acquired varies by space and demand process (Figure 1 (a)-(d)) and there is a high correlation between the numbers of buyers acquired through each process (Table 1). This underscores that in order to accurately measure the demand effects by the focal variables, our model must control for the regional baseline effects by each mode and their correlations. The coefficient of variation is higher for the WOM processes than for keyword search and magazine advertising, suggesting that WOM acquisitions are relatively more "concentrated".[5] That is, WOM does not take hold at every location, but when it does, it delivers customers disproportionately. Given that an expanded geographical market is a "two-edged sword" for Internet retailers—a boon for potential market size but a challenge for effective customer acquisition—it is critical to develop insight into what underlies these differences.

[Insert Table 1 about here]

It is important to note that during the data period Childcorp.com did no locally targeted marketing. There is no locally targeted spending for online search, and magazine advertising exposure is entirely determined by magazine subscriptions that are beyond this firm's control. Even though magazine circulation data are unavailable, our model controls for spatial variation through model random effects and the specification error (see Empirical Model section).

[5] To examine this more formally, we compute the Getis-Ord G^* statistic (Getis and Ord 1992). G^* statistics are higher for both WOM acquisitions than they are for online search and magazine advertising, supporting the observation that WOM acquisitions are locally concentrated.

We create the model dataset by using the zip code indicator to match firm's data with four other data sources: (1) the 2000 US Census, (2) local sales tax rate schedules, (3) shipping times, and (4) the 2007 US Census of Business and Industry. Our research therefore sheds some light on the applicability of widely-available "old economy" data from secondary sources to understanding retail institutions in the "new economy".

Focal Variables: Inter-Customer Proximity and Price and Convenience Benefits

Inter-Customer Proximity. Our interest centers on the incremental effect due to customer density—in particular, whether a heightened probability of customers coming into contact with each other has an especially large effect on acquisitions via WOM. Following Bell and Song (2007) and Choi, Hui, and Bell (2010) we create a measure of the physical proximity among potential customers. Target customers are households with children aged less than six years old; hence inter-customer proximity is the number of these households per square mile measured using the 2000 US Census data. (Our model also controls for the absolute number of such households and therefore market potential.)

[Table 2 about here]

Price Benefit. Childcorp.com prices are the same in every zip code, but the *relative online price advantage* varies across zip codes in accordance with local sales tax rates (Anderson et al. 2009; Goolsbee 2000). We compiled zip level sales tax rates using publicly available information from the Department of Revenue in each state and supplemented this with our own primary data. Since some states have sales tax exemptions for Childcorp.com products, we had to undertake an exhaustive manual check of local tax rates. We made over 1,000 telephone calls to a random sample of major retailers including Wal-Mart, Walgreens, and CVS and asked store employees to determine whether Childcorp.com products are tax exempt and requested that they verify their answer by physically scanning individual items. In approximately 17% of zip codes Childcorp.com does not benefit from a *de facto* tax advantage, either because no sales tax is assessed in these zip codes, or because the firm has a physical presence in the state. The average rate in zip codes where sales taxes are collected on Childcorp.com products is about 6.7%. The substantial spatial variation in zip level tax rates (Figure 2) suggests that these truly disaggregate data are needed to accurately measure the price benefit.

[Insert Figure 2 about here]

Convenience Benefit. Following Brynjolfsson and Smith (2000) we measure time distance with shipping time data. Shipping times are exogenously determined by the distance between the shopper's zip code and the Childcorp.com warehouses on the east and west coasts. Shoppers are informed about shipping days when they place orders. Expected shipping days are shown in Figure 3. The black region in the center of the US has four-day shipping (the base case in the empirical model). Relative to this base case, other areas in the east and west receive one, two, or three-day shipping.

[Insert Figure 3 about here]

Following Forman, Ghose, and Goldfarb (2009), travel distance is computed from local retail variables constructed from the 2007 US Census of Business and Industry. We use 8-digit NAICS (North American Industry Classification System) codes to obtain retail information about major local offline competitors, including supermarkets, discount stores (Wal-Mart and Target), and warehouse clubs.[6] Physical distance to stores parallels transportation costs in spatial differentiation models (e.g., Balasubramanian 1998; Bhatnagar and Ratchford 2004), so we

[6] While 6-digit NAICS codes are often used in research, greater accuracy is achieved with our approach. For example, 6-digit NAICS codes for supermarkets include candy stores and other smaller retail formats that differ from what is typically thought of as a supermarket. These NAICS codes have exact correspondence with SIC codes.

calculate the distance from the focal zip code to the nearest offline store of each format to capture the expected convenience of offline alternatives.

Control Variables and Spatial Clustering of Zip Codes

Geo-Demographic Characteristics.[7] We include a standard set of zip level control variables including measures of age, income, ethnicity, and education constructed from the US Census. Following Dhar and Hoch (1997) these variables are expressed as percentages, e.g., "percentage of households with a college degree." The overall potential market size is measured by the number of households with children less than six years of age and serves as an offset variable (Agresti 2002; Greene 2008).

Spatial Clustering of Zip codes. The US Census Bureau groups zip codes into MSAs (Metropolitan Statistical Areas) and μSAs (Mircopolitan Statistical Areas) on the basis of strong social and economic ties.[8] Zip codes in the same MSA or μSA share average characteristics, so we define regional clusters of zip codes using these designations; zip codes which do not belong to MSAs or μSAs are grouped by states. There are 358 MSAs and 567 μSAs in the 48 contiguous states. Regional cluster random effects are used to efficiently capture the difference in baselines across regional clusters.

[7] There is no significant multicollinearity among these variables. We thank the anonymous reviewer for suggesting this check. The largest pair-wise correlation is .48 and most pair-wise correlations are less than .30. Also, the largest VIF (variance inflation factor) is 4.06 in the regression model of count data in log form.

[8] MSAs are formed around a central urbanized area, i.e., a contiguous area of relatively high population density, and surrounding areas which have "strong ties" (as measured by commuting and employment) to the central area. Likewise, μSAs consist of adjacent areas that have at least one urban cluster. This spatial demarcation is more comprehensive than one based on geographical boundaries alone. Delaware Valley, for example, is a metropolitan area comprising several counties in Delaware, Maryland, New Jersey, and Pennsylvania.

EMPIRICAL MODEL

Given that the zip level customer acquisition numbers are non-negative integer counts, we model them in a Poisson framework. We assume that $y_{k,z(m)}$, the number of new buyers acquired by process *k* in zip code *z* in regional cluster *m*, is Poisson distributed.

(1) $$y_{k,z(m)} \sim \text{Poisson}(\lambda_{k,z(m)})$$

where *k* = offline WOM, online WOM, online search, and magazine advertising. We justify our modeling choice on both theoretical and empirical grounds. First, the Poisson is widely applied in spatial models when the occurrence of an event is rare in comparison with the target population (Choi, Hui, and Bell 2010; Wikle and Hooten 2006; Knorr-Held and Besag 1998), as is the case here. Second, in Appendix I we outline a mathematical argument (adapted from Berry 1994 and Blum and Goldfarb 2006) that the Poisson approximation for zip level counts can be motivated from individual level utility maximization decisions. Aggregation over individuals to a zip level count is also important as individual level covariates are unavailable; aggregation also helps cancel any errors in individual level recollection of the acquisition method. Third, the Poisson model is flexible enough to accommodate the spatial variation in baseline rates, correlations among the demand processes, and specification error in the dependent variable (more details are given below). The inclusion of cross-sectional heterogeneity in the Poisson model leads naturally to the negative binomial model (see equation 5 and Appendix I). Finally, we show in the next section that our model provides an excellent fit to the data and very good predictive accuracy in holdout samples.

The rate parameter $\lambda_{k,z(m)}$ is modeled as a function of: (1) inter-customer proximity, (2) price benefit, (3) convenience benefit, (4) the number of target customers or market potential, (5) a set of control variables for zip code observed heterogeneity, (6) unobserved baseline by regional cluster, and (7) zip level measurement error.

(2) $$\log(\lambda_{k,z(m)}) = x'_{k,z(m)}\beta_k + \varepsilon_{k,z(m)} \text{ and}$$

(3) $$\begin{aligned} x'_{k,z(m)}\beta_k &= \varphi_k \cdot TargetCustomerProximity_{z(m)} \\ &+ \Gamma_k \cdot PriceBenefit_{z(m)} + \Delta_k \cdot ConvenienceBenefit_{z(m)} \\ &+ \log(n_{z(m)}) + \alpha_{k,0} + \alpha_{k,m} + \Psi_k \cdot Controls_{z(m)} \end{aligned}$$

where $TargetCustomerProximity_{z(m)}$ measures target customer density, i.e., inter-customer proximity and φ_k is the corresponding scalar parameter. $PriceBenefit_{z(m)}$ has two variables, one for the presence of local sales tax and the other for the conditional tax rate, and Γ_k is the parameter vector for the effects. $ConvenienceBenefit_{z(m)}$ includes two sets of variables. The first set covers time convenience and includes six dummies for one, two, and three-day shipping on the east and west coasts, relative to the four-day benchmark. The second set has three variables for distances for the nearest offline competitor formats—supermarket, discount store, and warehouse club, i.e., travel distance. Δ_k is the parameter vector for the effects of convenience benefit. $n_{z(m)}$ is the number of target customers (households with children less than six years of age) and enters the model in log form to serve as an offset variable (Agresti 2002; Greene 2008; Rabe-Hesketh and Skrondal 2005).[9] $Controls_{z(m)}$ contains the measures for zip level observed heterogeneity (age, income, education, etc.) and Ψ_k is the corresponding parameter vector.

Considerable spatial variation in the raw data implies that we must carefully control for the regional baselines to accurately estimate the effects of the focal variables. To this end, the

[9] Using the number of target customers as an offset variable is justified in two ways. First, this approach is standard when the number of buyers is very small compared the size of the potential customers, which is supported by our data (see Tables 1 and 2). Second, the offset can be derived from individual level utility maximization decisions made by these same households (see Appendix I). Moreover, the natural log form for the offset variable is justified as it is the canonical link for the Poisson distribution.

baseline for regional cluster *m* comprises the overall baseline, $\alpha_{k,0}$, and the random deviation of regional cluster *m* from the overall baseline, $\alpha_{k,m}$. Furthermore, since all four demand processes emerge from the same regional cluster *m*, we let the four random effects follow a multivariate normal distribution in accordance with standard approaches for modeling clustered data (Gueorguieva 2001; Thum 1997).

$$(4)\qquad \begin{pmatrix} \alpha_{offlineWOM,m} \\ \alpha_{onlineWOM,m} \\ \alpha_{Search,m} \\ \alpha_{Magazine,m} \end{pmatrix} \sim \text{i.i.d. } MVN\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_1^2 & r_{21}\tau_2\tau_1 & r_{31}\tau_3\tau_1 & r_{41}\tau_4\tau_1 \\ r_{21}\tau_2\tau_1 & \tau_2^2 & r_{32}\tau_3\tau_2 & r_{42}\tau_4\tau_2 \\ r_{31}\tau_3\tau_1 & r_{32}\tau_3\tau_2 & \tau_3^2 & r_{43}\tau_4\tau_3 \\ r_{41}\tau_4\tau_1 & r_{42}\tau_4\tau_2 & r_{43}\tau_4\tau_3 & \tau_4^2 \end{pmatrix} \right)$$

This multivariate random effects approach has several benefits. First, it allows us to model the four acquisition modes simultaneously and also accommodate a variety of nested cases. (We can therefore demonstrate the superiority of this approach over the restricted nested models using fit comparison and holdout tests.) Second, since all four acquisition modes are modeled as a function of the same focal and control variables and the parameters are jointly estimated, we can directly compare the separate effects of one specific variable, e.g., online price benefit, across the demand outcomes. Finally, the multivariate model offers better control over the Type I error rates in multiple tests, and improves the efficiency of the parameter estimates.

The model also implicitly recognizes that the mode-specific numbers of new buyers per zip code could be an imperfect reflection of the true acquisition process at the individual level. It is possible, for example, that some buyers fail to indicate their true acquisition modes because of imperfect memory, or that some buyers are exposed to multiple sources of influence, but answer with the one mode that is most salient or most relevant.[10] We account for the specification error

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[10] The firm's use of a drop-down menu and mutually exclusive and exhaustive alternatives is deliberate as asking customers to assign weights to multiple sources of influence is problematic and potentially increases non-response

in the zip level dependent variable by the disturbance term, $\varepsilon_{k,z(m)}$, and assume that $\exp(\varepsilon_{k,z(m)})$ is independently and identically Gamma distributed with shape and scale parameter, θ_k (equal scale and shape parameters are needed for identification; see Cameron and Trivedi 1986; Greene 2008). The density for $y_{k,z(m)}$ after integrating out over $\exp(\varepsilon_{k,z(m)})$ becomes one form of the negative binomial distribution (NBD) with mean $\mu_{k,z(m)}$ and variance $\mu_{k,z(m)}\left(1+\theta_k^{-1}\mu_{k,z(m)}\right)$, and is given by

(5) $$f(y_{k,z(m)} \mid x_{k,z(m)}) = \frac{\Gamma(\theta_k + y_{k,z(m)})}{\Gamma(y_{k,z(m)}+1)\Gamma(\theta_k)} r_{k,z(m)}^{y_{k,z(m)}} (1-r_{k,z(m)})^{\theta_k}$$

where $\mu_{k,z(m)} = \exp(x'_{k,z(m)}\beta_{k,z(m)})$ and $r_{k,z(m)} = \dfrac{\mu_{k,z(m)}}{\mu_{k,z(m)}+\theta_k}$ (see equation (I.7) in Appendix I for the derivation). Note that the addition of the specification error $\varepsilon_{k,z(m)}$ also allows the variance of the dependent data to be larger than the mean, and a test of the Poisson assumption is given by $\theta_k^{-1} = 0$. The model in equation (5) has a closed form up to the random effects so likelihood is evaluated via numerical integration over the random effects. This evaluation becomes more computationally demanding as the dimension of the random effects vector increases. To circumvent this problem we follow Fieuws and Verbeke (2006) and Fieuws et al. (2006) and fit all pair-wise bivariate models separately and then calculate the parameter estimates and their sampling variation for the full multivariate model (see Appendix II). In

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rates. Moreover, “pick any” data are notoriously difficult to analyze (even four categories yields 15 possible response combinations). A single individual level choice coupled with aggregation to a zip level count and combined with explicit model specification error gives a better account of the data (aggregation helps cancel out individual level misclassification of self-reported acquisition mode). We thank an anonymous reviewer for prompting this observation.

addition, since we have only the likelihoods for the pair-wise models, we obtain the multivariate model likelihood through Monte Carlo sampling.

EMPIRICAL FINDINGS

Model Fit, Validation, and Spatial Autocorrelation Test

Model fits and validation results for the multivariate NBD model and the four nested models are given in Table 3. The log-likelihood of the multivariate model is significantly larger than those for the nested models. To ensure that this model is not over-fitting the data, we perform predictive validation using holdout tests. Since the data are cross-sectional and there is no natural ordering of observations, we conduct holdout tests by performing 10-fold cross validation on each combination of the estimation and validation datasets (Brieman and Spector 1992; Kim et al. 2005). Mean absolute errors for the multivariate model in the estimation and validation datasets are smaller than those from the nested models. The superior fits and holdout test results suggest that we must account for the random effects and their correlations (failure to do so could lead to biased estimates and potentially incorrect inference).

[Insert Table 3 about here]

To check that there is no remaining spatial autocorrelation in the residuals of the multivariate model, we compute Moran's *I* statistics using a spatial weighting matrix based on an exponential distance decay function (Moran 1950).[11] The Moran's *I* values are very small and statistically insignificant, which indicates that there is no remaining unaccounted for spatial autocorrelation.

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[11] The pair-wise weight between zip code *i* and zip code *j* is an exponential function of the inverse distance in miles, d_{ij}, and equal to exp(-Δd_{ij}). We further assume Δ is one. The latter assumption is made for computational tractability and consistency with prior work (e.g., Claude 2002; LeSage and Pace 2005; Yang and Allenby 2003).

Estimates for Inter-Customer Proximity and Price and Convenience Benefits

Table 4 reports the estimation results from the multivariate NBD model. The signs and significance levels for the model variables are consistent with our expectations. Note that inter-customer proximity, travel distance, and control variables are standardized and the parameters for a single covariate from the multivariate model are directly comparable across the four outcome variables. The final column of Table 4 reports the estimates from a model in which the dependent variable is the total customer count per zip code , $y_{z(m)} = \sum_k y_{k,z(m)}$, i.e., no distinction is made as to the acquisition mode. By comparing the estimates from this model to those from the multivariate NBD model that decomposes demand by the four separate processes we highlight insights that are unavailable from a simple aggregate analysis of sales.

[Insert Table 4 about here]

Inter-Customer Proximity. Target customer density has a positive and significant effect on demand through all four processes. The average lift for the total number of customers induced by a one standard deviation increase in this variable is 5.6%. However this masks the differential effects that flow through the different acquisition modes. Specifically, the effect of inter-customer proximity is significantly larger for WOM processes (7.9% for offline WOM and 7.1% for online WOM), and the two effects are not significantly different from each other. Conversely, the estimates for online search and magazine advertising of 3.5% and 4.1% respectively, are significantly lower than 5.6%, but not different from each other. At the individual level WOM is an interdependent process which involves endogenous feedback among buyers and potential buyers, and hence, it is facilitated by close physical proximity among buyers (see also Choi, Hui, and Bell 2010; Yang and Allenby 2003).

The quantitative effect of a change in customer density can be assessed not only in percentage terms (as above), but also in terms of the change in the expected number of new buyers. If we select 100 zip codes that have values of all the model variables at their means, this yields 131.5 expected new buyers in total. The expected total breaks down into 36.3 offline WOM buyers, 7.9 online WOM buyers, 38.0 search buyers, and 49.3 magazine buyers. Increasing customer density by one standard deviation brings 6.8 additional buyers, which are decomposed into 2.9 offline WOM, 0.6 online WOM, 1.3 online search, and 2.0 magazine advertising buyers. What is interesting is while WOM buyers account for only about one third of the initial pool of buyers, they account for half of the lift in new buyers that comes from a change in inter-customer proximity. This insight is unavailable from analysis of total demand only. It is potentially valuable to a firm utilizing different acquisition strategies across different locations.

Price Benefit. Controlling for the presence of sales tax, the effect of saving on sales tax is positive and statistically significant for WOM buyers and for search buyers. Prior research shows that search buyers are motivated by price (Bakos 1997; Lal and Sarvary 1999), but there is no comparable finding for WOM buyers. One conjecture is that price benefits could be salient in WOM conversations.[12] The fact that magazine advertising by Childcorp.com did not stress the online price benefit and that this price benefit has no significant effect on magazine advertising acquisitions suggests that the salience of product attributes (price, convenience, service) can be influenced by firm messaging. This finding is consistent with Lynch and Ariely (2000) who show that online shoppers can be made sensitive to quality as well as to price, if quality differences between alternatives are emphasized.

[12] One posting at Bizrate.com, for instance, emphasizes the importance of price and convenience benefits from shopping at Childcorp.com. “Great Internet Service!!! The shipping is very fast and there’s no tax. Especially loved free shipping with purchase of $49 or more. I have recommended to many of my friends already and our baby isn't even born yet. :)”

The estimates imply that a one percent increase in the sales tax rate will increase the number of new buyers by 5.4%, 4.3%, and 4.9% for offline WOM, online WOM, and online search, respectively. The average zip level sales tax rate is 6.7%, so if Childcorp.com buyers were taxed at this rate or, alternatively, offline sales were made tax exempt, the number of buyers shopping at Childcorp.com would decline. The same 100 zip codes discussed earlier that generate 131.5 expected buyers would lose about 32 of them. Almost half of the expected loss (15.2 buyers) would come from offline WOM buyers, even though these buyers account for a little less than one third of the initial total. Hence, everything else being equal, the firm is less able to generate buyers through offline WOM in zip codes that do not charge sales tax. Again, these nuanced insights have potentially important implications for firm activities and are unavailable from an analysis of total demand.

Convenience Benefit. Faster shipping has positive and significant effects as consumers respond favorably to superior time convenience, however, the overall effect varies dramatically by acquisition process. Once again, the average effects based on a model of the total number of new buyers mask important differences across the sub-processes. While all acquisition modes show strong absolute effects, fast shipping is especially effective in generating new buyers through offline WOM. Prior research shows that waiting time is a critical determinant of service satisfaction and therefore likely to be a key driver of offline WOM (Berry, Seiders, and Grewal 2002; DiClemente and Hantula 2003; Rose, Meuter, and Curran 2005). Moreover, the marginal improvement for each process is non-linear. The fact that one day and two day shipping, in particular, produce such strong effects might explain why many online retailers categorize shipping speeds as “one-day shipping”, “two-day shipping” and “standard shipping” (3-5 business days).

Like time distance, travel distance is also important, but not to all types of buyers. Childcorp.com products are carried by supermarkets, discount stores, and warehouse clubs. Of the three formats, supermarkets are the most prevalent offline competitors and the most convenient for shoppers (see Table 2). In interpreting the effects, it is important to keep in mind that Childcorp.com has a price advantage relative to the most numerous competitors (supermarkets) and is at price parity with discount stores. The estimate for “distance to the nearest supermarket” is significantly negative for offline WOM, online search, and magazine advertising. This implies that when they are *more* convenient, online sales go up. We offer two explanations for this seemingly counterintuitive effect of travel distance to supermarkets. First, shoppers who live closer to supermarkets shop more frequently (Bell and Lattin 1998) and are therefore likely to have superior price knowledge for individual product categories (Dinesh, Sudhir, and Talukdar 2008). This makes the online price advantage of Childcorp.com more salient and the site more attractive. Second, when shoppers travel farther to supermarkets they buy larger baskets of items (including products sold by Childcorp.com) in order to amortize fixed travel costs. Hence, when the offline supermarket is further away, travel cost amortization over the basket makes the online store less attractive. Travel distance for discount stores and warehouse clubs shows the more typical substitution effect reported elsewhere in the literature (Forman, Ghose, and Goldfarb 2009). Unlike supermarkets, these formats are price-competitive with Childcorp.com. Hence, when low-priced offline retail alternatives for products sold at Childcorp.com are relatively inconvenient, shoppers go online.

While the signs of the travel distance coefficients are identical across all four demand processes, the effects are markedly weakest for online WOM (only one of three coefficients is significant). Buyers acquired via online search and magazine advertising make independent assessments of shopping online, and consider travel distances to offline options. Buyers acquired via offline WOM experience online shopping benefits identical to those enjoyed by the sender, due to buyer co-location. Buyers acquired via online WOM may have received WOM from senders in distant markets; therefore, they do not experience convenience benefits that are the same as those enjoyed by the sender and they are relatively unaffected by travel convenience to offline options.

Summary. The decomposition of demand into separate acquisition processes generates nuanced insights unavailable from analysis of aggregate new buyer demand. As expected, WOM acquisitions generated by communication at the individual level have statistically larger demand coefficients at the zip code level. Offline and online WOM acquisitions show different patterns of response—for offline WOM senders and receivers are co-located, whereas for offline WOM they need not be. This produces larger effects, on average, of favorable local conditions on new buyer acquisition via offline WOM. Marginal effects imply that disaggregate analysis will have economically important implications for the firm for resource allocation to different customer acquisition methods at different types of locations.

Parameter Estimates for Control Variables and Variances

Control Variables. Estimates for geo-demographic control variables are not of direct interest, yet their signs and magnitudes are reassuring. Higher population growth rates, more individuals of child-bearing age, and factors such as higher percentages of college-educated individuals that increase the probability of shopping online in general, have positive effects.

Variances. The variance parameter for the disturbance term, θ_k, is significant across all four acquisition modes and variation unaccounted-for by model variables is largest for offline WOM (the variance increases in θ_k^{-1}, see Table 4). The variance and correlation terms for the multivariate random effects, τ_k^2 and $r_{kk'}$ ($k \neq k'$), are also significant across all four acquisition modes. Variance in the random effects implies that it could be worthwhile for the firm to uncover regional clusters with higher baselines, along with local zip codes that exhibit high sensitivity to local factors. Positive correlations in acquisition mode baselines suggest that if a regional cluster has a larger propensity to generate buyers through one mode of acquisition, this cluster also brings forth new buyers through the other three. Interestingly, this estimated correlation is very strong between offline WOM and the other three acquisition modes. This highlights the “special” nature of offline WOM and underscores why prominent practitioners refer to it as the “world’s most effective marketing strategy” (see Trusov, Bucklin, and Pauwels 2009 for a discussion).

Expected Demand by Space and Acquisition Mode

New Buyer Decomposition. The model provides a novel view of demand performance by space and acquisition mode. To demonstrate we compute the expected number of buyers per zip code and sort all 29,652 from best to worst. For ease of illustration, we then create 50 groups of zip codes so that each group has roughly an equal number of total expected buyers (roughly 3,300 per group). High performing groups have far fewer zip codes while low performing groups have many more. Group 1, for example, has only four zips and Group 2 twenty-one zips, while Group 49 has 3,417 and Group 50 has 11,630.

Figure 4 shows the percentage decomposition of buyers by acquisition mode on the *y*-axis plotted against the group number, 1-50, on the *x*-axis. Offline WOM accounts for two-thirds of the total expected buyers in Group 1 but its relative contribution drops off rapidly to one fourth in Group 50. As expected from the estimation results, the best performing groups of zip codes tend to have larger values for the focal variables. The average values of target customer density, for example, decrease quickly from 4,000 in Group 1 and 260 in Group 25 to 4.2 in Group 50, and the average offline sales tax rates are about 7.5% and 5.5% in Group 1 and Group 50, respectively. Magazine advertising acquisitions show a reverse pattern—they start at around 15% and increase to over 40% of the total expected buyers. The reciprocal patterns of the two traditional acquisition methods reveal their complementary nature in overall customer growth. Offline WOM is especially effective in locations that are particularly "fertile," whereas magazine advertising has considerably more "reach" into many regions of demand that have lower individual potential, but collectively still account for a sizable portion of the customer base. Online search acquisitions show a similar but less pronounced increasing pattern—from 13% to 28% of total expected new buyers—and online WOM acquisitions are a constant contribution of about 6% in all groups of zips.

[Insert Figure 4 about here]

Figures 5 (a) and (b) complement the information in Figure 4 by placing model-based expected acquisitions on a physical map of the US. Each zip code is highlighted according to the acquisition mode that is most effective there, for zip codes that have at least one expected buyer (Figure 5 (a)) and ten expected buyers (Figure 5 (b)). No one acquisition mode dominates spatially as different markets favor one over the others. However some strong empirical patterns emerge. Offline WOM dominates in a small number of very high performing spatially-clustered zip codes, whereas traditional magazine advertising is very effective in reaching broad geographical markets that correspond to low-performing groups that individually generate few buyers, but are collectively substantial. In Figure 5 (a) there are many green zip codes, i.e., zip codes where magazine advertising generates the most expected new buyers. However among "high performing zip codes" in Figure 5 (b), there are relatively few green regions. In summary, it is clear that while IS-enabled methods are vital for increasing the "reach" of the customer base, traditional methods of acquisition remain significant, even in the new economy. Since Internet retailers have different ways of attracting customers, this suggests local customization of acquisition strategies could be worthwhile.[13]

[Insert Figure 5 about here]

Preliminary Evidence for Geo-Targeting. Bronnenberg and Albuquerque (2003) suggest that more needs to be done to "… analyze the observed differences in within-firm marketing strategy across markets." Hence, we offer one final analysis that highlights a practical implication of the results. An important managerial question is: What decisions should the firm make differently in light of the findings from our model? We answer this by showing how the firm might think about the locally-customized purchase of search keywords. Search engines charge for sponsored links on a cost-per-click basis and while it is now possible to purchase search keywords on a geographical basis, Childcorp.com has never done this. To explore the potential of this option, we examine improvements that could result from locally-targeted search keywords, where promising local targets are identified by the model.

We obtain conversion rates from "first click" to "first order" among first-time visitors at Childcorp.com for about 1,200 major cities in the United States, from October 2007 through

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[13] As noted in the Data section, Childcorp.com did no locally-targeted marketing with any acquisition method. Childcorp.com or other Internet retailers could however employ locally-adjusted acquisition strategies. Out of the four modes, online search is directly under the firm's control and the firm can locally adjust search spending. Magazine subscriptions are beyond the firm's control, but Childcorp.com could distribute advertisements through local newspapers. Online WOM can be promoted through bloggers and online brand communities established via social networking sites. Finally, Childcorp.com can facilitate offline WOM by using referral incentives or by supporting local moms' communities (see also Godes and Mayzlin (2009) for a discussion of firm-initiated WOM). We thank an anonymous reviewer for these suggestions.

March 2008, from Coremetrics.com.[14] We then compare *actual conversion* (i.e., click-to-order), with the predictions of *potential* coming from our model. To do this, we first use the model to generate an overall prediction for the total number of new buyers for each zip code. We use total-number predictions because: (1) new buyers are likely to access Childcorp.com via search engines regardless of their initial acquisition mode, and (2) there is no acquisition-mode-specific conversion rate data available (Coremetrics.com does not provide this information).

Next, we aggregate zip code predictions in each of the 1,200 major cities in the Coremetrics.com database, and sort the cities from highest to lowest according to the expected number of new buyers per household in the target population. After this sorting, we form 50 separate groups of cities from the initial pool of the major cities. The fifty groups of cities are defined so that each group has roughly equal numbers of new clicks (i.e., roughly equal marketing costs) and cities in each group have similar "predicted performance" (i.e., numbers of new buyers per number of households with children aged less than siz years old).

[Insert Table 5 about here]

For the sake of brevity, Table 5 shows results for only six (out of fifty) groups of cities: the top two, middle two, and bottom two groups. The key columns are column (5) for the model-based prediction of new buyers per household with children and column (6) for the *actual* click-to-order conversion rates captured by Coremetrics.com. Top groups of cities have conversion rates of about 18-19% and need, on average, 5.5 new clicks to obtain one new buyer. This increases to ten and twelve clicks for the middle and bottom groups, respectively. By targeting groups of cities with good model-based expected performance, the firm can improve efficiency

[14] Our data are at the zip level, whereas Coremetrics.com data are at the city level. Coremetrics.com specializes in tracking visitor browsing and purchasing behavior at online sites, for visitors coming from major US cities. They started collecting data for Childcorp.com management from October 2007. The number of new buyers in these major cities accounts for 52% of the total new buyers, despite the relatively small number of cities included.

in click-through rates by a factor of about 2. This preliminary evidence from completely separate conversion information suggests that leveraging predictions from a spatial model based on “old economy” geo-demographic data could yield economically meaningful improvements in marketing expenditures on keywords.[15]

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

An online retailer is by definition ubiquitous—shoppers almost anywhere have the *potential* to use it—however, the propensity for them to do so varies by geographical location. Specifically, it is well known that online retail demand at a given location is explained by physical proximity among target customers as it promotes social influence (e.g., Choi, Hui, and Bell 2010), and also by the relative price and convenience of offline alternatives (e.g., Forman, Ghose, and Goldfarb 2009). Much less is known, however, about how online retailers attract customers to begin with. That is, while online firms attract customers from heterogeneous locations using traditional and IS-enabled methods (Figure 1), the efficacy of these methods as a function of location has not been investigated. This is an important area for research given both the vast trading area of Internet retailers and the economic cost-benefit of using different acquisition approaches. Using a multivariate NBD model, we explain customer counts in zip codes as function of local characteristics and uncover important differences across acquisition methods. Our main empirical findings are as follows.

- *WOM Acquisitions Benefit from Physical Proximity among Target Customers*. We find statistically stronger effects of target density for customers acquired through WOM as

[15] Coremetrics.com data provides two additional insights. Individuals in cities with good model-based expected performance: (1) click more pages per session and (2) stay longer at Childcorp.com per session, suggesting that they are more engaged with Childcorp.com. We thank the anonymous reviewer for suggesting this interesting check.

WOM is an inherently interdependent process at the individual consumer level, and therefore benefits disproportionately from factors that engender direct conversation or indirect observation. Marginal effects from the model estimates imply that while WOM customers account for one third of the total customers in “average” zip codes, they account for one half of the lift in customers induced by an increase in inter-customer proximity.

- *Relative Benefits Still Matter—Especially to WOM Customers*. Online demand responds to relative online price and convenience benefits. There are, however, important nuances of difference by acquisition mode. Price and time convenience effects are especially strong for offline WOM customers. It is plausible that such benefits are part of the WOM conversation. Magazine advertising customers who are exposed to a service-oriented message are not influenced by price considerations. This is consistent with the finding that price and quality sensitivity of online retail customers is malleable (Lynch and Ariely 2000). Offline WOM customers who are co-located with WOM senders and therefore experience the same benefits of shopping online, are sensitive to offline travel distances, whereas online WOM customers are not.
- *Acquisition Mode Efficacy Varies Dramatically over Space*. Model-based expected demand by acquisition mode is highlighted in Figures 4 and 5. A key insight, unavailable from analysis of total demand alone, is that each mode shows a dramatically different pattern over space. Traditional methods remain significant in a complementary manner for retailers in the new economy. Offline WOM is a highly effective acquisition mode in a smaller number of spatially clustered and high performing zip codes. The relative effectiveness of magazine advertising as an acquisition mode increases in a greater number of “weaker” locations with small total demand. IS-enabled acquisition methods of online search and

online WOM contribute a relatively constant percentage to the customer base over all locations.

Our findings raise new questions that suggest fruitful avenues for future research. First, "old economy data" seem to be quite relevant to predicting sales in the new economy (as evidenced by our preliminary geo-targeting analysis). More work is needed to show how firms could benefit in an economically meaningful way by, for example, buying search keywords in locations that models identify as having high potential. It would also be worthwhile to test the effectiveness of specific modes in different markets. Second, one could examine how the firm could generate WOM (e.g., Godes and Mayzlin 2009), the product and service dimensions that should be stressed in any such effort, and the role of online forums in propagating the information (Dellarocas 2003). We intend to pursue these and other related topics in future research.

References

Anderson, Eric T., Nathan M. Fong, Duncan I. Simester, and Catherine E. Tucker (2009), "How Sales Taxes Affect Customer and Firm Behavior: The Role of Search on the Internet," *Journal of Marketing Research*, forthcoming.

Agresti, Alan (2002), *Categorical Data Analysis*, Wiley: New York, NY.

Bakos, Yannis (1997), "Reducing Buyer Search Costs: Implications for Electronic Marketplaces," *Management Science*, 43 (12), 1676–1692.

Balasubramanian, Sridhar (1998), "Mail versus Mall: A Strategic Analysis of Competition between Direct Marketers and Conventional Retailers," *Marketing Science*, 17 (3), 181-195.

Becker, G. and K. Murphy (2000), *Social Economics*. Cambridge, MA: Harvard University Press.

Bell, David R., and James M. Lattin (1998), "Shopping Behavior and Consumer Preference for Store Price Format: Why 'Large Basket' Shoppers Prefer EDLP," *Marketing Science*, 17 (1), 66-88.

________ and Sangyoung Song (2007), "Neighborhood Effects and Trial on the Internet: Evidence from Online Grocery Retailing," *Quantitative Marketing and Economics*, 5 (4), 361-400.

Berry, Leonard L., Kathleen Seiders, and Dhruv Grewal (2002), "Understanding Service Convenience," *Journal of Marketing*, 66 (3), 1-17.

Berry, Steven T. (1994), "Estimating Discrete-Choice Models of Product Differentiation," *Rand Journal of Economics*, 25 (2), 242-262.

Bhatnagar, Amit and Brian T. Ratchford (2004), "A Model of Retail Format Competition for Non-Durable Goods," *International Journal of Research in Marketing*, 21 (1), 39-59.

Blum, Bernardo B. and Avid Goldfarb (2006), "Does the Internet Defy the Law of Gravity?" *Journal of International Economics*, 70 (2), 384-405.

Breiman, Leo and Philip Spector (1992), "Submodel Selection and Evaluation in Regression: The X-Random Case," *International Statistical Review*, 60 (3), 291-319.

Bronnenberg, Bart J and Paulo Albuquerque (2003), "Geography and Marketing Strategy in Consumer Packaged Goods," *Advances in Strategic Management*, 20, 215-237, Elsevier Science, Amsterdam.

Brynjolfsson, Erik and Michael D. Smith (2000), "Frictionless commerce? A Comparison of Internet and Conventional Retailers," *Management Science*, 46 (4), 563–585.

Cameron, A. Colin and Pravin K. Trivedi (1986), "Econometric Models Based on Count Data: Comparisons and Applications of Some Estimators and Tests," *Journal of Applied Econometrics*, 1 (1), 29-54.

Claude, Besner (2002), "A Spatial Autoregressive Specification with a Comparable Sales Weighting Scheme," *Journal of Real Estate Research*, 24 (2), 193-211.

Choi, Jeonghye, Sam K. Hui, and David R. Bell (2010), "Spatio-Temporal Analysis of Imitation Behavior," *Journal of Marketing Research*, 47 (1), 1-15.

Dellarocas, Chrysanthos (2003), "The Digitization of Word-of-Mouth: Promise and Challenges of Online Feedback Mechanisms," *Management Science*, 49 (10), 1407-1424.

________ and Charles A. Wood (2008), "The Sound of Silence in Online Feedback: Estimating Trading Risks in the Presence of Reporting Bias," *Management Science*, 54 (3), 460-476.

Dhar, Sanjay and Stephen J. Hoch (1997), "Why Store Brand Penetration Varies by Retailer," *Marketing Science*, 16 (3), 208-227.

DiClemente, Diane F. and Donald A. Hantula (2003), "Optimal Foraging Online: Increasing Sensitivity to Delay," *Psychology & Marketing*, 20 (9), 785-809.

Dinesh K. Gauri, K. Sudhir, and Debabrata Talukdar (2008), "The Temporal and Spatial Dimensions of Price Search: Insights from Matching Household Survey and Purchase Data," *Journal of Marketing Research*, 45 (2), 226-240.

Fieuws, Steffen and Geert Verbeke (2006), "Pairwise Fitting of Mixed Models for the Joint Modeling of Multivariate Longitudinal Profiles," *Biometrics*, 62 (2), 424-431.

________, ________, Filip Boen, and Christophe Delecluse (2006), "High Dimensional Multivariate Mixed Models for Binary Questionnaire Data," *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 55 (4), 449-460.

Foreman, Chris, Anindya Ghose, and Avi Goldfarb (2009), "Competition Between Local and Electronic Markets: How the Benefit of Buying Online Depends on Where You Live," *Management Science*, 55 (1), 47-57.

Fotheringham, Stewart A. (1988), "Consumer Store Choice and Choice Set Definition," *Marketing Science*, 7 (3), 299-310.

Getis, Arthur, and J. Keith Ord (1992), "The Analysis of Spatial Association by Use of Distance Statistics," *Geographical Analysis*, 24, 189-206.

Glaeser, Edward L., Bruce Sacerdote, and Jose A. Scheinkman (1996), "Crime and Social Interactions," *Quarterly Journal of Economics*, 111 (2), 507-548.

Godes, David and Dina Mayzlin (2009), "Firm-Created World-of-Mouth: Evidence from a Field Test," *Marketing Science*, 28 (4), 721-739.

Goolsbee, Austan (2000), "In a World Without Borders: The Impact of Taxes on Internet Commerce." *Quarterly Journal of Economics*, 125, 561-576.

Greene, William (2008), *Econometric Analysis*, Prentice Hall: Upper Saddle River, NJ.

Gueorguieva, Ralitza (2001), "A Multivariate Generalized Linear Mixed Model for Joint Modeling of Clustered Outcomes in the Exponential Family," *Statistical Modeling*, 1, 177-193.

Huff, D. (1964), "Redefining and Estimating a Trading Area," *Journal of Marketing*, 28 (3), 34-38.

Katz, Elihu and Paul F. Lazarsfeld (1955), *Personal Influence*, Glencoe, IL: Free Press.

Keeney, Ralph L. (1999), "The Value of Internet Commerce to the Customer," *Management Science*, 45 (4), 533-542.

Kim, YongSeong, W. Nick Street, Gary J. Russell, and Filippo Menczer (2005), "Customer Targeting: A Neural Network Approach Guided by Genetic Algorithms," *Management Science*, 51 (2), 264–276.

Knorr-Held, Leonhard and Julian Besag (1998), "Modeling Risk from a Disease in Time and Space," *Statistics in Medicine*, 17 (18), 2045-2060.

Lal, Rajiv and Miklos Sarvary (1999), "When and How Is the Internet Likely to Decrease Price Competition?" *Marketing Science*, 18 (4), 485-503.

LeSage, J. P. and R. K. Pace (2005), "Matrix Exponential Spatial Specification," *Journal of Econometrics*, 140 (1), 190-214.

Lodish, Leonard M., Magid Abraham, Stuart Kalmenson, Jeanne Livelsberger, Beth Lubetkin, Bruce Richardson, and Mary Ellen Stevens (1995), "How T.V. Advertising Works: A Meta-Analysis of 389 Real World Split Cable T.V. Advertising Experiments," *Journal of Marketing Research*, 32 (2), 125-139.

Lynch, John G. and Dan Ariely (2000), "Wine Online: Search Costs and Competition on Price, Quality, and Distribution," *Marketing Science*, 19 (1), 83-103.

Moran, P.A.P. (1950), "Notes on Continuous Stochastic Phenomena," *Biometrika*, 37 (12), 17–33

Rabe-Hesketh, Sophia and Anders Skrondal (2005), *Multilevel and Longitudinal Modeling Using Stata*, Statacorp LP: College Station, TX.

Rose, Gregory M., Matthew L. Meuter, and James M. Curran (2005), "On-line Waiting: The Role of Download Time and Other Important Predictors on Attitude Toward e-Retailers," *Psychology & Marketing*, 22(2), 127–151.

Tang, Christopher S., David R. Bell, and Teck-Hua Ho (2001), "Store Choice and Shopping Behavior: How Price Format Works," *California Management Review*, 43 (2), 56-65.

Thum, Yeow Meng (1997), "Hierarchical Linear Models for Multivariate Outcomes," *Journal of Educational and Behavioral Statistics*, 22 (1), 77-108.

Trusov, Michael, Randolph E. Bucklin, and Koen Pauwels (2009), "Effects of Word-of-Mouth Versus Traditional Marketing: Findings from an Internet Social Networking Site," *Journal of Marketing*, 73 (5), 90-102.

Villanueva, Julian, Shijin Yoo, and Dominique M. Hanssens (2008), "The Impact of Marketing-Induced vs. Word-of-Mouth Customer Acquisition on Customer Equity," *Journal of Marketing Research*, 45 (1), 48-59.

Wall Street Journal (2007), "Keeping Baby Dry: We Buy Bulk Diapers," http://www.wsj.com.

Wikle, Christopher K. and Mevin B. Hooten (2006), "Hierarchical Bayesian Spatio-Temporal Models for Population Spread," In Clark, J.S. and A. Gelfand (Eds.) *Applications of Computational Statistics in the Environmental Sciences: Hierarchical Bayes and MCMC Methods*, Oxford University Press, USA.

Yang, Sha and Greg M. Allenby (2003), "Modeling Interdependent Consumer Preferences," *Journal of Marketing Research*, 40 (3), 282-294.

Table 1

Numbers of New Buyers per Acquisition Mode per Zip Code

Acquisition Process[1]	**Mean**	**Standard Deviation**	**Sum**	**Correlations**[2]		
				Offline WOM	**Online WOM**	**Online Search**
Buyers from Offline Word-of-Mouth	1.829	7.945	54,246	---		
Buyers from Online Word-of-Mouth	.347	1.132	10,300	.786	---	
Buyers from Online Search	1.422	3.328	42,170	.818	.757	---
Buyers from Magazine Advertising	1.743	3.386	51,681	.686	.685	.802
Total Buyers	5.342	14.501	158,397			

Notes

[1] Zip code penetrations by acquisition mode are as follows: Buyers from offline word-of-mouth (11,689 zips), Buyers from online word-of-mouth (5,716 zips), Buyers from online search (12,261 zips), Buyers from magazine advertising (13,978 zips). There are 29,652 residential zip codes in the database. 18,244 of these zip codes (about 62%) have at least one buyer.

[2] All the correlations are significantly different from zero ($p < .01$).

Table 2

Summary Statistics for Zip Code Level Model Covariates

Variable	**Mean**	**Standard Deviation**	**Min**	**Max**
Target Customer Proximity				
Density of Households with Children ≤ 6 Years Old	65.890	256.242	0	7398.909
Price Benefit				
No Tax = 1 if No Tax is Levied in Zip Code	.171	.377	0	1
Local Sales Tax Rate (%)[1]	6.655	1.186	2.900	9.750
Convenience Benefit – Time Distance				
Days to Ship to Zip Code	2.624	.967	1	4
Convenience Benefit – Travel Distance				
Distance to the Nearest Supermarket	4.064	4.366	0	65.029
Distance to the Nearest Discount Store	13.105	13.921	0	180.283
Distance to the Nearest Warehouse Club	31.760	33.316	.044	332.644
Control Variables				
Number of Households with Children ≤ 6 Years Old	562.525	850.750	0	9705
Growth Rate in Number of Households (2000-2004)	.013	.018	-.126	.337
Percentage Population Aged 20 to 39 Years Old	.258	.068	0	.868
Percentage Households with Working Female	.032	.051	0	1
Percentage of Whites	.850	.198	0	1
Percentage of Blacks	.076	.157	0	.985
Percentage with College Education	.452	.163	0	1
Percentage Households Earning $50,000-$75,000	.188	.059	0	1
Percentage Households Earning $75,000-$150,000	.188	.059	0	1
Percentage Households Earning $150,000 or more	.142	.093	0	1

Notes

[1] Summary statistics for the local sales tax rate are computed across 24,573 residential zip codes that have local sales taxes on Childcorp.com products.

Table 3

Model Fit Comparison: Proposed Model and Nested Models

Model	**Specification**	**Log-Likelihood**	**MAE[1] (Mean absolute error)**	
			In-sample	**Out-of-sample**
Proposed model	NBD model with multivariate random effects	-115,801	.811	.832
Nested model 1	NBD model with univariate random effects ($r_{kk'}$ = 0 for all k and k', $k \neq k'$, in equation (4))	-116,129	.844	.880
Nested model 2	NBD model with no random effects ($\alpha_{k,m}$ = 0 for all k and m, in equation (3))	-117,284	.911	.918
Nested model 3	NBD model with no random effects ($\alpha_{k,m}$ = 0), holding the parameter vector for control variables (Ψ_k) constants across four modes (k's) in equation (3)	-118,344	.931	.934
Nested model 4	NBD with model no random effects ($\alpha_{k,m}$ = 0), holding all parameters (φ_k, Γ_k, Δ_k, and Ψ_k) constant across four modes (k's) in equation (3)	-118,832	.945	.952

Notes

[1] We conduct holdout tests by performing 10-fold cross validation on each partition of the estimation and validation datasets (Brieman and Spector 1992; Kim et al. 2005). Estimation and validation datasets include 26,687 and 2,965 residential zip codes, respectively.

Table 4

Parameter Estimates[1]

Variables	Multivariate NBD Model[2]								Total Buyers[3]	
	Offline WOM		Online WOM		Online Search		Magazine Ads			
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Target Customer Proximity										
φ, Density, HH with Children Aged ≤ 6 Yrs	.076**	.011	.068**	.009	.035**	.009	.040**	.005	.055**	.005
Price Benefit										
Γ_1, No Tax Dummy	.142	.197	.151	.155	.186	.140	.045	.111	.127	.085
Γ_2, Local Sales Tax Rate (%)	.052*	.024	.042*	.020	.048**	.018	.019	.014	.030**	.011
Convenience Benefit – Time Distance										
Δ_1, One-Day Shipping, Eastern US	1.185**	.111	.740**	.156	.857**	.095	.744**	.094	.890**	.062
Δ_2, Two-Day Shipping, Eastern US	.553**	.070	.315**	.079	.378**	.051	.408**	.048	.431**	.044
Δ_3, Three-Day Shipping, Eastern US	.322**	.056	.218**	.059	.244**	.044	.291**	.040	.290**	.039
Δ_4, One-Day Shipping, Western US	.664**	.131	.441**	.110	.460**	.091	.292**	.075	.460**	.085
Δ_5, Two-Day Shipping, Western US	.285**	.078	.137*	.072	.204**	.059	.037	.053	.150**	.055
Δ_6, Three-day Shipping, Western US	.025	.093	-.138	.100	-.017	.068	-.049	.053	-.095	.061
Convenience Benefit – Travel Distance										
Δ_7, Distance to Nearest Supermarket	-.082**	.019	-.059	.032	-.064**	.016	-.076**	.015	-.076**	.010
Δ_8, Distance to Nearest Discount Store	.271**	.030	.172**	.030	.237**	.019	.196**	.019	.230**	.012
Δ_9, Distance to Nearest Warehouse Club	.127**	.021	.061	.033	.053**	.018	.140**	.017	.097**	.013
Control Variables										
α_0, Model Intercept	-7.052**	.177	-8.317**	.162	-6.840**	.134	-6.294**	.103	-5.448**	.081
Ψ_1, Growth Rate in Number of HH	.184**	.025	.132**	.030	.186**	.020	.194**	.018	.196**	.006
Ψ_2, Percent Population Aged 20 to 39 Years	.155**	.031	.165**	.033	.101**	.027	.063**	.020	.099**	.009
Ψ_3, Percent HH with Working Female	.011	.039	.008	.043	-.040	.024	.012	.020	.000	.013

Ψ_4, Percent with College Education	.600**	.035	.492**	.041	.470**	.029	.356**	.023	.455**	.012
Ψ_5, Percent of Whites	.337**	.058	.265**	.052	.227**	.033	.311**	.038	.239**	.016
Ψ_6, Percent of Blacks	.087	.056	.057	.041	.057	.033	.052	.034	.036	.014
Ψ_7, Percent HH Earning $50K-$75K	-.031	.030	.026	.029	.027	.020	.051*	.021	.008	.011
Ψ_8, Percent HH Earning $75K-$150K	-.151**	.035	-.118**	.044	-.159**	.028	-.067**	.025	-.116**	.013
Ψ_9, Percent HH Earning $150K or more	.079**	.015	.054**	.016	.009	.011	.028*	.012	.056**	.009
Variances										
θ [4]	2.478**	.122	2.933**	.259	4.709**	.258	5.403**	.271	2.731**	.042
τ	.381**	.025	.254**	.023	.267**	.019	.215**	.014	.329**	.017
r_{21} (Online WOM, Offline WOM)							.990**	.034		
r_{31} (Online Search, Offline WOM)							.971**	.012		
r_{32} (Online Search, Online WOM)							.768**	.194		
r_{41} (Magazine Ads, Offline WOM)							.967**	.010		
r_{42} (Magazine Ads, Online WOM)							.688**	.188		
r_{43} (Magazine Ads, Online Search)							.808**	.069		

Notes

[1] For each estimate, we test the null hypothesis that the parameter is equal to zero. Statistical significance is indicated by * which shows significance at $p < .05$ and ** which shows significance at $p < .01$.

[2] The dependent variable is the number of new buyers acquired through each process in each zip code and all the variables except those for local sales tax and time distance are standardized (see equations (1)-(4)).

[3] The dependent variable is the total number of new buyers aggregated over the four processes in each zip code and all the variables except those for local sales tax and time distance are standardized.

[4] The variance of $y_{k,z(m)}$ is $\mu_{k,z(m)}\left(1+\theta_k^{-1}\mu_{k,z(m)}\right)$. A test of θ_k^{-1}=0 versus θ_k^{-1}>0 is a test of the Poisson versus NBD and all four processes favor NBD models.

Table 5

A Comparison of Model Predictions and Click-to-Order Conversions

Cities Per Group	HHs w/ Children (1)	Expected Buyers (2)	First Orders (3)	First Clicks (4)	Expected Buyers per HHs w/ Children (5)=(2)/(1)	Conversion Rates (6)=(4)/(3)
Top Two Groups						
1	67,098	5,194	9,924	54,119	.077	.183
21	80,260	2,405	2,260	11,904	.030	.190
Middle Two Groups						
46	228,172	2,154	1,133	10,673	.009	.106
16	208,026	1,914	1,013	11,207	.009	.090
Bottom Two Groups						
42	394,773	1,816	886	10,942	.005	.081
44	252,416	976	905	10,946	.004	.083

Notes

Each group of cities has about 11,000 of clicks (i.e., roughly equal marketing costs) and all cities in a group have roughly equal predictions for the expected number of new buyers per household. The best performing group contains one city, New York City. The number of cities in the other groups is variable. In the interests of space, we show only six groups of cities and indicate the differences between the “best” (top two), “average” (middle two) and “worst” (bottom two) groups of cities. Full information for all 50 groups is available from the authors upon request.

Figure 1

Spatial Distribution of New Buyers per Acquisition Method per Zip Code

	Traditional Acquisition Methods	IS-Enabled Acquisition Methods
Customer-generated Acquisitions per Zip Code (*Interdependence* at the individual consumer level)	**(a) Offline Word-of-Mouth**	**(b) Online Word-of-Mouth**
Firm-initiated Acquisitions per Zip Code (*Independence* at the individual consumer level)	**(c) Magazine Advertising**	**(d) Online Search**

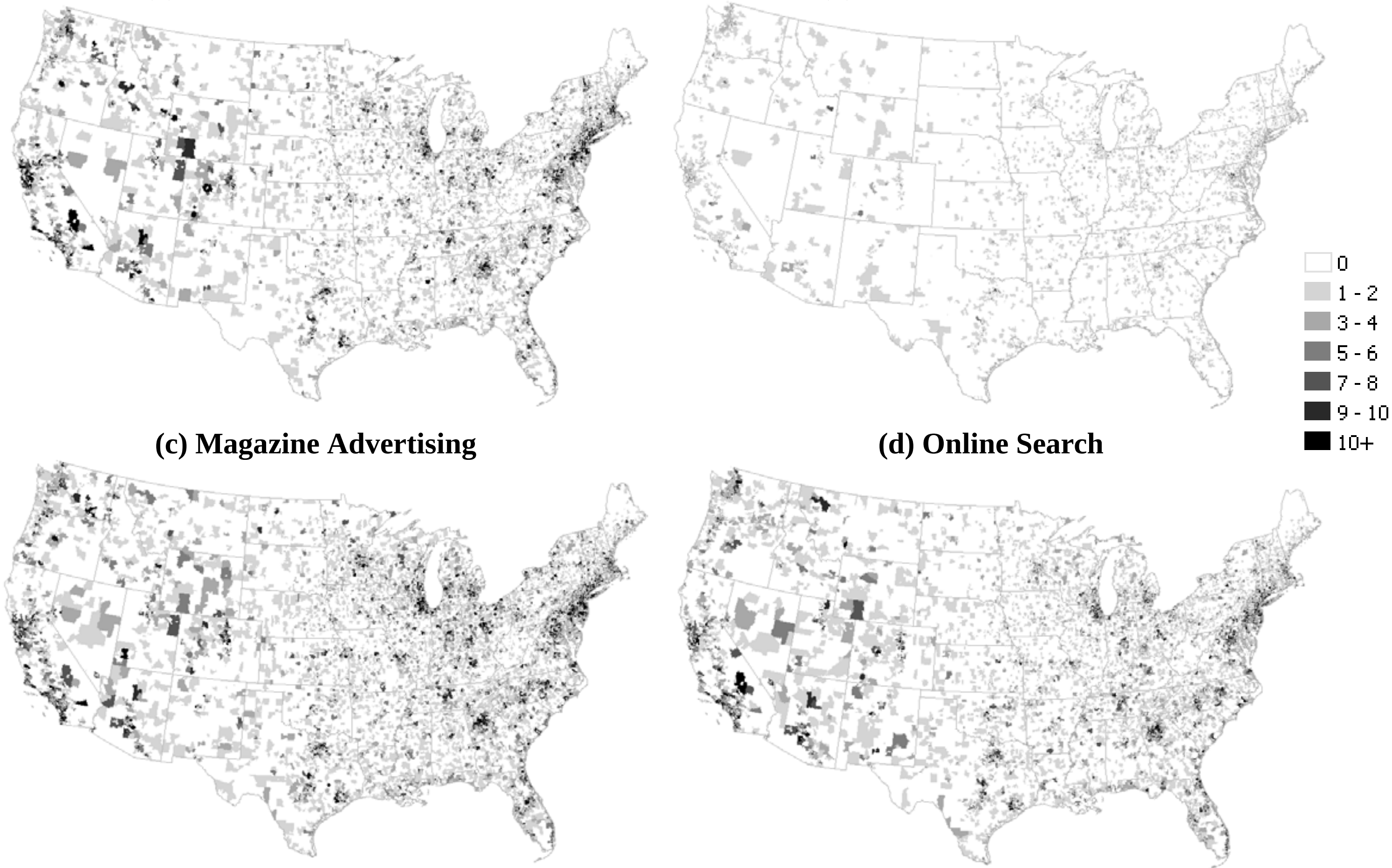


Figure 2

Spatial Distribution of Offline Sales Tax Rates (in Percentages)

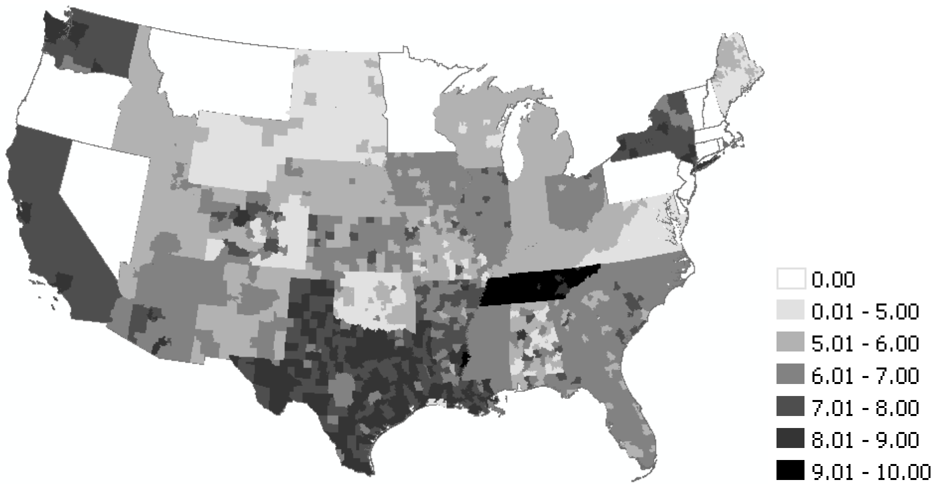


Figure 3

Spatial Distribution of Shipping Times (in Days)

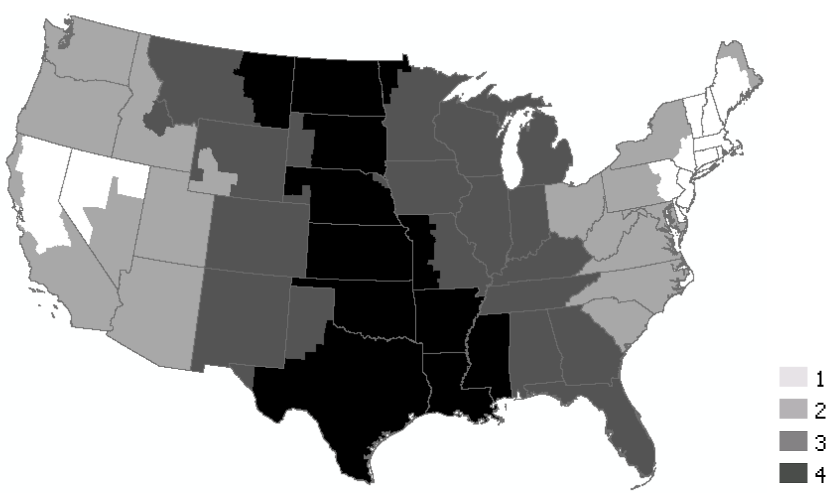
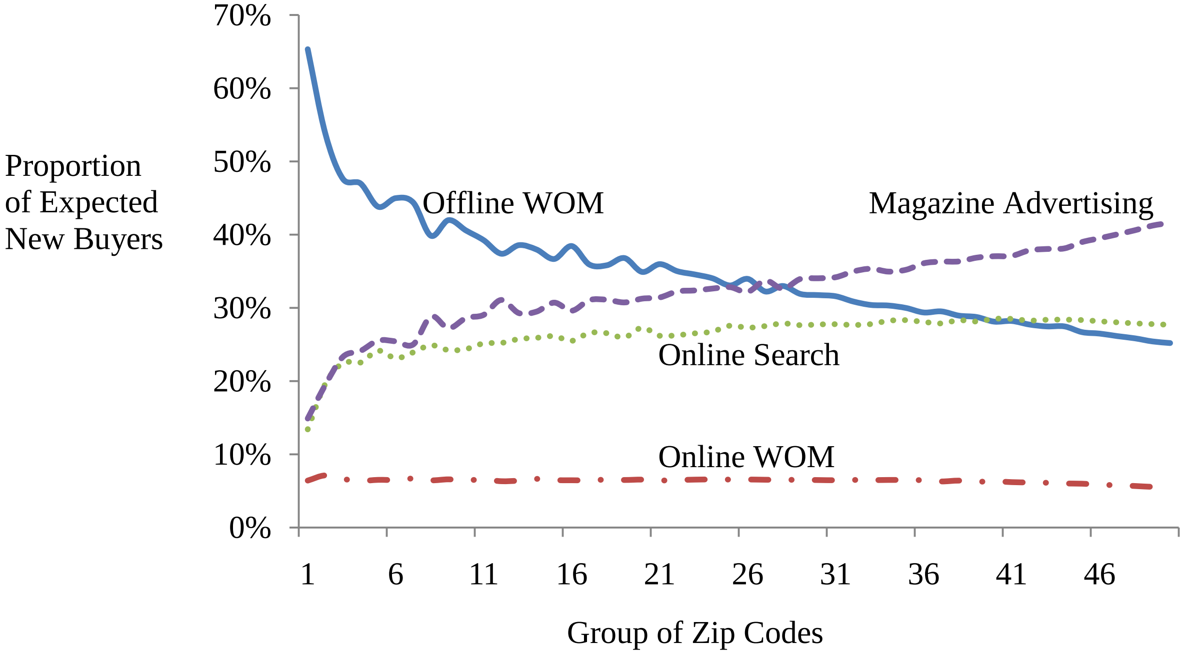


Figure 4

The Mix of New Buyers through Traditional and IS-Enabled Acquisition Modes



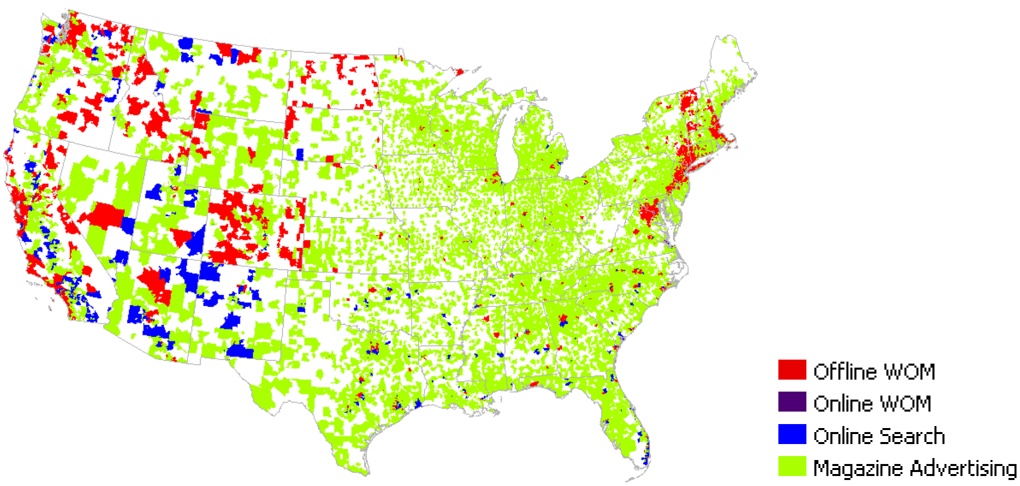
Notes

The fifty groups of zip codes are defined so that each group has roughly equal numbers of expected buyers (about 3,300 buyers per group). The number of zip codes per group declines from four in Group 1 to 11,630 in Group 50.

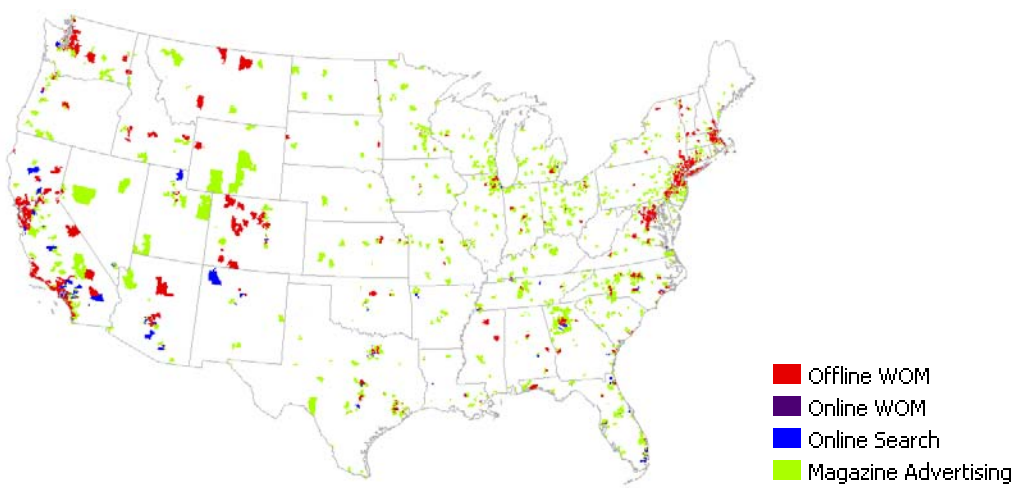
Figure 5

Spatial Distribution of the Most Effective Acquisition Process in Each Zip Code

(a) Zip Codes with More Than One Expected Buyer



(b) Zip Codes with More Than Ten Expected Buyers



Notes

The colors indicate which mode is most effective in each location, i.e., which mode generates the greatest expected number of new buyers.

APPENDIX

I. Poisson Model

Relationship between Individual level Decision Making and Aggregate Demand

To motivate the econometric approach, we develop a model of aggregate online demand, adapted from Berry (1994) and Blum and Goldfarb (2006). Since we focus on households with children as target customers, the disaggregate demand for Childcorp.com by each target customer is derived from utility maximization. We then aggregate household level decisions to the zip code level, enabling analyses using zip level data.

Suppose the utility that a target customer *i* in zip code z gets from buying Childcorp.com products from retailer *r* can be represented as

(I.1) $$u_{i,rz} = x'_{rz}\beta + \varepsilon_{rz} + v_{i,rz}$$

where x_{rz} is a vector of observed characteristics of retailer *r* and zip code *z*, ε_{rz} captures unobserved characteristics of retailer *r* and zip code *z*, and $v_{i,rz}$ represent a target customer *i*'s unobserved characteristics. Assume that $v_{i,rz}$ is type two extreme value distributed. This implies that the market share of retailer *r* in zip code *z* is given by

(I.2) $$s_{rz} = \frac{\exp(x'_{rz}\beta + \varepsilon_{rz})}{1 + \sum_{w} \exp(x'_{rz}\beta + \varepsilon_{rz})}$$

where *w* denotes all the retailers selling Childcorp.com products.

Since the parameters of the model covariates are identified up to scale, we let the "non-purchase" choice (i.e., not buying Childcorp.com products from any retailer available) be the outside choice. Let s_{oz} represent the market share of the non-purchase choice. Then, the difference between s_{rz} and s_{oz} is given by a linear function of observed characteristics of retailer *r* and zip code *z*.

(I.3) $$\log(s_{rz}) - \log(s_{oz}) = x'_{rz}\beta + \varepsilon_{rz}$$

If we write s_{rz} as the ratio of the number of buyers for retailer *r* in zip code *z*, y_{rz}, and the total number of target customers in zip code z, n_z, then we obtain the following.

(I.4) $$\log(y_{rz}) = \log(n_z) + \alpha_z + x'_{rz}\beta + \varepsilon_{rz}$$

$\alpha_z = \log(s_{oz})$ might be computed from the secondary data or estimated using zip level fixed or random effects. This approach is similar in spirit to Bell and Song (2007) who link individual level utility maximization to a region level discrete time hazard model with a complementary log-log link function.

Poisson Framework for Aggregate Count Data

Equation (I.4) is an approximation to the standard Poisson model for one particular retailer with the number of target customers in zip code z, n_z, as an offset variable. Using the number of target customers as an offset variable makes sense when we focus on one retailer out of many alternatives selling products to the target group, as is the case for our data. Here we model the number of new buyers at Childcorp.com in zip code *z*, y_z, in a Poisson framework and model the rate parameter, λ_z, as a function of the offset variable, observed characteristics of the firm and zip code *z*, and the disturbance term, ε_z.

(I.5) $$y_z \sim \text{Poisson}(\lambda_z),$$

(I.6) $$\log(\lambda_z) = \log(n_z) + x_z'\beta + \varepsilon_z$$

In (I.6) β is redefined with an intercept shifted by $\alpha_z = \log(s_{oz})$ in equation (I.4). We assume $\exp(\varepsilon_z)$ to be independently and identically Gamma-distributed with shape and scale parameter, θ (equal scale and shape parameters are needed for identification; see Cameron and Trivedi 1986; Greene 2008). Then, the density for y_z after integrating out over the disturbance, becomes one form of the negative binomial distribution with mean μ_z and variance $\mu_z\left(1+\theta^{-1}\mu_z\right)$, and is given by

(I.7)
$$\begin{aligned} f(y_z \mid x_z) &= \int_0^\infty \frac{e^{-\mu u}(\mu u)^y}{y!} g(u)du \\ &= \int_0^\infty \frac{e^{-\mu u}(\mu u)^y}{y!} \frac{\theta^\theta u^{\theta-1} e^{-\theta u}}{\Gamma(\theta)} du \\ &= \frac{\Gamma(\theta + y_z)}{\Gamma(y_z+1)\Gamma(\theta)} r_z^{y_z}(1-r_z)^\theta \end{aligned}$$

where $\mu_z = \exp(x_z'\ \beta)$, $u_z = \exp(\varepsilon_z)$, and $r_z = \dfrac{\mu_z}{\mu_z + \theta}$.

II. Pair-wise Estimation for the Multivariate Acquisition Model

The dimensionality of the random effect vector makes estimation computationally difficult. To alleviate the high computational demand, we implement a pair-wise modeling approach to obtaining unbiased and efficient estimates. Here we outline the approach; interested readers are encouraged to see Fieuws and Verbeke (2006) and Fieuws et al. (2006) for more details.

Random Effects for the Multivariate Model

Let $y_{z(m)} = \{y_{1,z(m)}, y_{2,z(m)}, y_{3,z(m)}, y_{4,z(m)}\}^T$ be a vector containing the numbers of new buyers acquired by offline WOM, online WOM, online search, and magazine advertising, in zip code *z* in regional cluster *m* ($z = 1, \cdots, n_m$ and $m = 1, \cdots, N$). Let $y_m = \{y_{1m}, \cdots, y_{n_m m}\}^T$ denote the vector of all observations within the m^{th} cluster. Let $\alpha_m = \{\alpha_{1,m}, \alpha_{2,m}, \alpha_{3,m}, \alpha_{4,m}\}^T$ be a vector containing the random effects for offline WOM, online WOM, online search, and magazine advertising, in regional cluster *m*. Conditional on α_m, $y_{z(m)}$ of y_m are assumed to be independent. Inference is based on the marginal model for y_m after integrating out the random effects over the distribution of α_m.

Pair-wise Estimation

Let Θ^* be the vector containing all fixed effect parameters and variance parameters.

Step 1. Fit pair-wise models: Let Θ_{ij} be the vector of all parameters in the pair of the i^{th} and j^{th} dependent variables. We obtain parameter estimates for Θ_{ij}, $\hat{\Theta}_{ij}$, separately for each pair. Within a maximum likelihood framework, each bivariate model yields consistent estimates with classical asymptotic properties.

Step 2. Inference for Θ. Let Θ be the stacked vector combining all pair-wise parameter vectors for Θ_{ij}. An asymptotic multivariate normal distribution for $\hat{\Theta}$ is derived from the pseudo-likelihood framework.

(II.1) $$\sqrt{N}\left(\hat{\Theta} - \Theta\right) \sim MVN\left(0, J^{-1}KJ^{-1}\right)$$

where *J* is a block-diagonal matrix with diagonal blocks J_{pp}, and *K* is a symmetric matrix containing blocks K_{pq}, with *p*, *q* = 1,…,6. These blocks are given by

(II.2) $$J_{pp} = -\frac{1}{N}\sum_{m=1}^{N} E\left(\frac{\partial^2 l_{pm}}{\partial\theta_p \partial\theta_p^T}\right)$$

(II.3) $$K_{pq} = -\frac{1}{N}\sum_{m=1}^{N} E\left(\frac{\partial l_{pm}}{\partial\theta_p}\frac{\partial^2 l_{qm}}{\partial\theta_p^T}\right)$$

Step 3. Inference for Θ^*. Estimates for Θ^* are obtained by $\hat{\hat{\Theta}}^* = A\Theta$ where A is a matrix containing the appropriate coefficients to calculate the average $\hat{\Theta}^*$ from $\hat{\Theta}$. $\hat{\Theta}^*$ follows a multivariate normal distribution with mean Θ^* and covariance matrix $A\left(N^{-1}(J^{-1}KJ^{-1})\right)A^T$.